$$\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{n+5^n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{n+1}}{n+5^n} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 2 \cdot 2^n}{n+5^n}$$

$$n+5^n > 5^n$$

$$\frac{1}{n+5^n} < \frac{1}{5^n}$$

$$\frac{2^n}{n+5^n} < \frac{2^n}{5^n}$$

$$\uparrow$$

$$\left(\frac{2}{5}\right)^n \qquad \sum \left(\frac{2}{5}\right)^n$$

geometric series

$r = \frac{2}{5}$

thus converges

absolutely convergent by Direct Comparison Test

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n - \ln n}$$

$$n - \ln n < n$$

$$\frac{1}{n - \ln n} > \frac{1}{n} \quad \text{and} \quad \ln n > 1$$

$$\frac{\ln n}{n - \ln n} > \frac{1}{n}$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by harmonic series

So by Direct Comparison Test

the original does <u>not</u> converge absolutely

$$f(x) = \frac{\ln x}{x - \ln x} \qquad f'(x) = \frac{\frac{1}{x}(x - \ln x) - (1 - \frac{1}{x})(\ln x)}{(x - \ln x)^2} = \frac{1 - \frac{\ln x}{x} - \ln x + \frac{\ln x}{x}}{(x - \ln x)^2}$$

$$= \frac{1 - \ln x}{(x - \ln x)^2}$$

$$1 - \ln x < 0$$
$$-\ln x < -1$$
$$\ln x > 1$$
$$x > e^1$$

$$\lim_{x \to \infty} \frac{\ln x}{x - \ln x} = \lim_{x \to \infty} \frac{\frac{\ln x}{\ln x}}{\frac{x}{\ln x} - \frac{\ln x}{\ln x}} = \lim_{x \to \infty} \frac{1}{\frac{x}{\ln x} - 1} = \lim_{x \to \infty} \frac{1}{\frac{1}{\frac{1}{x}} - 1} =$$

$$\lim_{x \to \infty} \frac{1}{x - 1} = 0$$

thus original converges Conditionally by the alternating series test

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by harmonic series thus not absolutely convergent.

$f(x) = \frac{1}{x}$ $\quad f'(x) = -\frac{1}{x^2} < 0$ $\quad \lim_{x \to \infty} \frac{1}{x} = 0$

thus the original is conditionally convergent by alternating series test.

$$\sum_{n=1}^{\infty} (-1)^n \left(\sqrt{n^2+n} - n\right)$$

$$\sum_{n=1}^{\infty} \sqrt{n^2+n} - n \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} = \sum_{n=1}^{\infty} \frac{n^2+n-n^2}{\sqrt{n^2+n}+n} = \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+n}+n}$$

$$= \sum_{n=1}^{\infty} \frac{\frac{n}{n}}{\frac{\sqrt{n^2+n}}{n} + \frac{n}{n}} = \sum_{n=1}^{\infty} \frac{1}{\frac{\sqrt{n^2+n}}{\sqrt{n^2}}+1} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\frac{n^2+n}{n^2}}+1} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$ thus diverges by the n^{th} term test

So the original diverges by the n^{th} term test

$$1+\frac{1}{4}-\frac{1}{9}-\frac{1}{16}+\frac{1}{25}+\frac{1}{36}-\frac{1}{49}-\frac{1}{64}+\cdots$$

So look at $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots = \sum_{n=1}^{\infty}\frac{1}{n^2}$

Which converges by p-series thus the original will converge absolutely.

recall if $\sum_{n=1}^{\infty}|a_n|$ converges then $\sum_{n=1}^{\infty}a_n$ converges absolutely.