MATH 125 College Algebra Name____________________________

FUNDAMENTAL THEOREM OF ALGEBRA
Every polynomial of positive degree with complex coefficients has at least one complex root.

Descartes' Rule of Signs, Let $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \ldots + a_{n-1}x^1 + a_n$ be a polynomial function with real coefficients:

- The number of positive real zeros is either equal to the number of sign changes of $f(x)$ or is less than the number of sign changes by an even integer.
- The number of negative real zeros is either equal to the number of sign changes of $f(-x)$) or is less than the number of sign changes by an even integer.

$f(x) = 2x^3 - 4x^2 + 5x - 10$ *Positve real zeros* *Negative real zeros* *Complex non-real zeros*

$f(-x) =$

$n =$

$f(x) = x^4 - 2x^3 - 5x^2 + 12x - 6$ *Positve real zeros* *Negative real zeros* *Complex non-real zeros*

$f(-x) =$

$n =$

$f(x) = x^6 - x^5 - 2x^4 + 3x^3 - x^2 + 6x + 5$ *Positve real zeros* *Negative real zeros* *Complex non-real zeros*

$f(-x) = x^6 + x^5 - 2x^4 - 3x^3 - x^2 - 6x + 5$

$n =$

Rational Root Theorem If $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \ldots + a_{n-1}x^1 + a_n$ has integer coefficients and $\frac{p}{q}$ (p *and* q *are relatively prime*) is a rational zero of f, then p is a factor of a_0 and q is a factor of a_n

$f(x) = 2x^3 - 4x^2 + 5x - 10$

$f(x) = x^4 - 2x^3 - 5x^2 + 12x - 6$

LINEAR FACTORIZATION THEOREM

If $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + \ldots + a_{n-1}x^1 + a_0$, $n \geq 1$ *and* $a_n \neq 0$ *then*

$$f(x) = a_0(x - c_1)(x - c_2)(x - c_3)(x - c_4) \ldots (x - c_n)$$

where $c_1, c_2, c_3, c_4, \ldots, c_4$ *are complex numbers* (*possibly real and not necessarily distinct*).

Find a polynomial function of least degree in factored form with integer coefficients that has the given roots and passes through the given point.

Roots: $2, \ -2, \ \frac{1}{2}$ Point: $(0, \ 20)$

Roots: $-5, \ 3i$ Point: $(3, \ -72)$

Roots: $4, \ \frac{3}{10}, \ 2-i$ Point: $(-1, \ 13)$

Use the graph to the right to help determine all the zeros of the function

$$p(x) = 6x^5 + 7x^4 - 165x^3 - 138x^2 + 928x - 288.$$