MATH 141 OpenStax Name____________________________

Section 3.4 Derivative as Rates of Change

Motion along a Line

Another use for the derivative is to analyze motion along a line. We have described velocity as the rate of change of position. If we take the derivative of the velocity, we can find the acceleration, or the rate of change of velocity. It is also important to introduce the idea of **speed**, which is the magnitude of velocity. Thus, we can state the following mathematical definitions.

Definition

Let $s(t)$ be a function giving the position of an object at time t.

The velocity of the object at time t is given by $v(t) = s'(t)$.

The speed of the object at time t is given by $|v(t)|$.

The acceleration of the object at t is given by $a(t) = v'(t) = s''(t)$.

Find the velocity and acceleration and where the object is slowing down and speeding up. $s(t) = 2t^3 - 3t^2 - 12t + 8$

Definition

If $C(x)$ is the cost of producing x items, then the **marginal cost** $MC(x)$ is $MC(x) = C'(x)$.

If $R(x)$ is the revenue obtained from selling x items, then the marginal revenue $MR(x)$ is $MR(x) = R'(x)$.

If $P(x) = R(x) - C(x)$ is the profit obtained from selling x items, then the **marginal profit** $MP(x)$ is defined to be $MP(x) = P'(x) = MR(x) - MC(x) = R'(x) - C'(x)$.

We can roughly approximate

$$MC(x) = C'(x) = \lim_{h \to 0} \frac{C(x+h) - C(x)}{h}$$

The Cost-Function, in dollars, of a company that manuafactures food processors is given by $C(x) = 200 + \frac{7}{x} + \frac{x^2}{7}$, where x is the number of food processors manufactured. Find the marginal cost function. Find the marginal cost of manufacturing 12 food processors. Find the actual cost of manufacturing the thirteenth food processor.

Definition

If $P(t)$ is the number of entities present in a population, then the population growth rate of $P(t)$ is defined to be $P'(t)$.

The Holling type II equation is described by $f(x)=\dfrac{ax}{n+x}$, where x is the amount of prey available and $a>0$ is the maximum consumption rate of the predator.

a. Graph the Holling type II equation given $a=0.5$ and $n=5$.

b. Take the first derivative of the Holling type II equation and interpret the physical meaning of the derivative.

c. Show that $f(n)=\dfrac{1}{2}a$ and interpret the meaning of the parameter n.

d. Find and interpret the meaning of the second derivative.